Quantum structures: processes, axioms, causality – Syllabus

What are the mathematical structures encapsulating the weirdness of quantum theory? How can they be formally pinned down to ease their handling? How, in particular, can the crucial notion of *causal structure* be salvaged within a theory that seems to disregard it in such a careless way? Can one also superpose causal influences? These are the questions that this course will answer. We will show how recently developed mathematics (based in particular on category theory) help us to reformulate quantum theory from the ground up as a *process theory*, by taking its peculiar properties (first and foremost entanglement) as axioms. This opens to a renewed perspective on the notion of causal structure. Going to *higher-order quantum processes* – processes that map a quantum evolution to another quantum evolution –, we will discover that even causal structures might be put in a superposition in a quantum computer, leading to new speedups and applications.

Prerequisites: linear algebra, basic quantum information, and a taste for abstraction. Having followed the P1 course “ZX calculus” is a plus but not required.

List of sessions

- Lecture 1 (2h) – Quantum theory as a process theory, symmetric monoidal categories, daggers, entanglement as cups and caps
  then TD 1 (1h) – categories of quantum processes, category of relations.

- Lecture 2 (2h) – Classical structures as Frobenius algebras, no-cloning, no-broadcasting, CJ isomorphism, quantum curryfication
  then TD 2 (1h) – Markov categories, quantum lambda calculus.

- Lecture 3 (2h) – Doubling, discarding, the causality condition, purification
  then TD 3 (1h) – purification in the alternative categorical model (rel,)

- TD 4 (3h) – Categorical axiomatisation of Hilb.

- Lecture 5 (2h) – OPTs and re-axiomatisations of quantum theory
  then TD 5 (1h) – Local tomography and real quantum theory

- Lecture 6 (2h) – Quantum causal models
  then TD 2 (1h) – Causal decompositions

- Lecture 7 (2h) – Higher-order quantum computation
  then TD 7 (1h) – The quantum if

- TD 8 (3h) – Properties of higher-order maps: purifications, type systems, causal structure